CERTAIN PROPERTIES OF MIXING LAYERS

E. V. Bogdanova and O. S. Ryzhov UDC 532.526

We are examining the properties of solutions of the Falkner–Skan equation in the limiting case when the parameter in the equation approaches zero. Two types of boundary conditions are formulated. The first type corresponds to flow in a symmetric wake. The second corresponds to flow about a plate. The results of calculations and an asymptotic analysis make it possible to conclude that the transition from one type of boundary conditions to another involves a sharp change in the position of the mixing layer.

1. Let x and y be Cartesian space coordinates, ν be kinematic viscosity, and U(x) be the longitudinal component of velocity on the outside edge of the boundary layer. Following Gertler [1], we introduce the dimensionless variables

$$\xi = \frac{1}{\nu}\int_0^x U(x)\,dx, \quad \eta = yU(x)\left[2\nu\int_0^x U(x)\,dx\right]^{-\frac{1}{2}} \tag{1.1}$$

and take the stream function ψ in the form

$$\psi = \nu\,(2\xi)^{1/2} f(\xi, \eta). \tag{1.2}$$

The equation for ψ appears as

$$\frac{\partial^3 f}{\partial\eta^3} + f\frac{\partial^2 f}{\partial\eta^2} + \beta(\xi)\left[1 - \left(\frac{\partial f}{\partial\eta}\right)^2\right] = 2\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta} - \frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right), \tag{1.3}$$

where

$$\beta = 2U^{-2}(x)\frac{dU}{dx}\int_0^x U(x)\,dx.$$

Let us formulate boundary conditions with $\eta = 0$ corresponding to flow in a symmetrical wake and flow about a flat plate. On the axis of the wake

$$f = \partial^2 f/\partial\eta^2 = 0, \tag{1.5}$$

while on the surface of the plate

$$f = \partial f/\partial\eta = 0. \tag{1.6}$$

The condition on the outside edge of the boundary layer is formulated as a limiting condition

$$\partial f/\partial\eta \to 1 \quad \text{at} \quad \eta \to \infty \tag{1.7}$$

for all the flows studied below.

When $U = U_0$ = const, the parameter $\beta = 0$. In this case, the function f may depend on a single variable η. However, except for the trivial solution $f = \eta$, Eq. (1.3) has no self-similar solution that would satisfy boundary conditions (1.5). Replacement of conditions (1.5) by (1.6) leads to the familiar problem of uniform flow about a semi-infinite plate. Its solution, belonging to Balusius [1], is not of interest for the subsequent analysis of boundary layers with reverse fluid flows. We will use the function f_0 obtained in [2] as the initial self-similar solution. This function satisfies the first of conditions (1.5) or (1.6), but instead of the second of these conditions — expressing the requirement of

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 45-54, May-June, 1986. Original article submitted March 14, 1985.

0021-8944/86/2703-0359$12.50

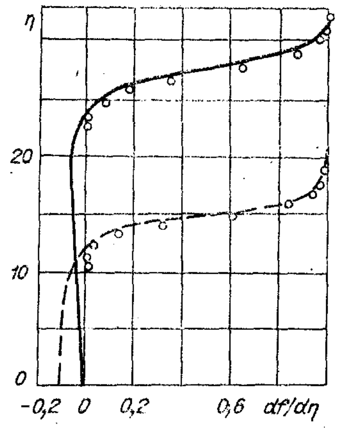

Fig. 1

symmetry of the flow or adhesion of the fluid to the surface of the solid – yet another asymptotic relation is satisfied

$$df_0/d\eta \to 0 \qquad \text{at} \quad \eta \to -\infty. \tag{1.8}$$

Chapman's solution describes a mixing layer which separate a uniform flow with the velocity U_0 in the top half-plane $y > 0$ from a quiescent fluid in the bottom half-plane $y < 0$.

2. Let x_0 be a certain constant with the dimension of length. If

$$U = U_0\left(1 + \frac{1}{2}\beta_0 \ln\frac{x}{x_0}\right), \quad |\beta_0| \ll 1,$$

then the velocity of the external potential flow will change slightly along the x axis except for the small neighborhood of the point $x = 0$. In accordance with (1.4), in the first approximation we find $\beta = \beta_0$. The sense of introducing independent variables of (1.1) is clear from this. The solution for the stream function in these variables remain self-similar and is determined from the ordinary differential equation

$$\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} + \beta_0\left[1 - \left(\frac{df}{d\eta}\right)^2\right] = 0. \tag{2.1}$$

When $\beta_0 \to 0_+$, the solution of Eq. (2.1) with boundary conditions (1.6) approaches the solution found by Blausius [1]. To construct a velocity field which includes the reverse fluid flow, it is necessary to put $\beta_0 \to 0_-$ and change over to another branch of the solution indicated in [3]. The solid curve in Fig. 1 shows results of calculation for $\beta_0 = -6.0 \cdot 10^{-4}$ which makes it possible to judge the qualitative properties of the asymptotic solution for flow about the plate.

Equation (2.1), with boundary conditions (1.5) was integrated numerically in [4] for the case of very small negative values of β_0. The dashed line in Fig. 1 shows the distribution of the horizontal component of velocity across the wake for the same value of $\beta_0 = -6.0 \cdot 10^{-4}$.

The above data shows that the asymptotic solution at $\beta_0 \to 0_-$ is accompanied by an infinite increase in the size of the region occupied by the reverse flow. Meanwhile, a mixing layer is formed on the external edge of this region. The mixing layer is adjacent to the incoming uniform flow. The circles in Fig. 1 pertain to the solution obtained in [2] for the above-mentioned three-point boundary-value problem for Eq. (2.1) with $\beta_0 = 0$. However, the coordinate system was shifted so that the point of inflection for the derivative $df/d\eta$ coincided with the analogous point in the solutions for a symmetrical wake and a boundary layer on a plate. The calculations leave no doubt that all of the mixing layers examined are identical. It is important to emphasize that although a change in the boundary conditions in the integration of (2.1) does not affect the internal structure of the mixing layer, it does entail an abrupt spatial shift. Given the same values of β_0, the Champion layer in the wake problem is located much lower than the same layer formed in flow about the plate.

Similar properties characterize the solution obtained in [5] for the problem of a plate in a uniform flow based on the assumption that there is no external pressure gradient. This solution is not self-similar. The region of recirculatory motion in it is described by the relation $\psi = -Hy^2$ with a positive constant H. An appreciably thinner mixing layer is located at the distance

$$y = (h/H)^{1/2}x^{1/4} \tag{2.2}$$

from the plate, the characteristic cross-sectional dimension of this layer increasing in proportion to $x^{1/2}$. The constant h fixes the rate of return flow in the mixing layer. The latter, as follows from (2.2), rises higher above the plate as H is arbitrarily reduced. Since the pressure gradient here is zero, the asymptotic solution is not connected with the properties of the external flow. The latter flow is accounted for by means of the parameter β_0 in Eq. (2.1). Thus, even very small changes in pressure along the boundary layer may have a significant effect on the structures of the velocity field in the recirculation region.

3. We will prove the above arguments by means of asymptotic analysis of Eq. (2.1). We use f_{0a} to designate the Champion solution f_0 shifted along the η axis by the amount a. As before, it satisfies asymptotic relations (1.7) and (1.8), but it does not satisfy the first of boundary conditions (1.5) or (1.6). When $\beta_0 \to 0_-$, we will seek the solution of Eq. (2.1) in the form

$$f = f_{0a}(\eta) + \varepsilon f'(\eta), \quad \varepsilon \to 0, \tag{3.1}$$

without making any assumptions regarding the relative value of the two small parameters. Linearization of Eq. (2.1) after insertion of Eq. (3.1) into it gives

$$\frac{d^3f'}{d\eta^3} + f_{0a}\frac{d^2f'}{d\eta^2} + \frac{d^2f_{0a}}{d\eta^2}f' + \frac{\beta_0}{\varepsilon}\left[1 - \left(\frac{df_{0a}}{d\eta}\right)^2\right] = 0. \tag{3.2}$$

In accordance with (1.7), we require that

$$df'/d\eta \to 0 \quad \text{at} \quad \eta \to \infty. \tag{3.3}$$

Following [6], we integrate Eq. (3.2), having made use of the fact that the homogeneous equation corresponding to it has two linearly independent integrals:

$$f_1' = \frac{df_{0a}}{d\eta}, \quad f_2' = f_{0a} + \eta\frac{df_{0a}}{d\eta}. \tag{3.4}$$

If we put

$$\eta = \xi + a, \tag{3.5}$$

then the initial solution is written as

$$f_{0a}(\eta) = f_0(\xi). \tag{3.6}$$

Proceeding on the basis of Eq. (3.4), we finally have

$$f = f_0(\xi) + \varepsilon A_1\frac{df_0}{d\xi} + \varepsilon A_2\left[f_0(\xi) + \xi\frac{df_0}{d\xi}\right] + \varepsilon A_3\frac{df_0}{d\xi}I_2(\xi) + \beta_0\frac{df_0}{d\xi}J_3(\xi). \tag{3.7}$$

Here, A_1, A_2, and A_3 are arbitrary constants, while the integral terms I_2 and J_3 are determined by means of the equalities

$$I_2(\xi) = \int_\xi^\infty\left[2 - \frac{f_0(\xi_1)\, d^2f_0/d\xi_1^2}{(df_0/d\xi_1)^2}\right]I_1(\xi_1)\, d\xi_1; \tag{3.8}$$

$$I_1(\xi_1) = \int_{\xi_1}^\infty \frac{df_0/d\xi_2 \times d^2f_0/d\xi_2^2}{[2(df_0/d\xi_2)^2 + d^3f_0/d\xi_2^3]^2}\, d\xi_2; \tag{3.9}$$

$$J_3(\xi) = \int_\xi^\infty\left[2 - \frac{f_0(\xi_1)\, d^2f_0/d\xi_1^2}{(df_0/d\xi_1)^2}\right]J_2(\xi_1)\, d\xi_1; \tag{3.10}$$

$$J_2(\xi_1) = \int_{\xi_1}^{\infty} \frac{df_0/d\xi_2 \times d^2f_0/d\xi_2^2}{\left[2\,(df_0/d\xi_2)^2 + d^3f_0/d\xi_2^3\right]^2} J_1(\xi_2)\,d\xi_2; \tag{3.11}$$

$$J_1(\xi_2) = \int_{\xi_2}^{\xi_0} \frac{\left[1-(df_0/d\xi_3)^2\right]\left[2\,(df_0/d\xi_3)^2 + d^3f_0/d\xi_3^3\right]}{d^2f_0/d\xi_3^2}\,d\xi_3. \tag{3.12}$$

It is henceforth necessary to know the asymptotic behavior of the solution when $\xi \to \infty$ and $\xi \to -\infty$, which depends on the corresponding asymptotes of the Champion function (the constant $\xi_0 > 0$ in the expression for J_1 is assumed to be sufficiently large). When $\xi \to \infty$, in accordance with boundary condition (1.7) we find

$$f_0 \sim \xi - b + B \int_{\xi}^{\infty} d\xi_1 \int_{\xi_1}^{\infty} e^{-\frac{(\xi_2-b)^2}{2}}\,d\xi_2 + \ldots, \tag{3.13}$$

while when $\xi \to -\infty$, asymptotic relation (1.8) leads to

$$f_0 \sim -c + c^{-2}C\,e^{c\xi} + \ldots \tag{3.14}$$

The values of the constants b, c, B, and C are calculated by joining both asymptotes with the solution for the region where $|\xi| \sim 1$. Insertion of expansion (3.13) into (3.9) permits us to conclude that the integral I_1 approaches zero exponentially if $\xi_1 \to \infty$. Returning to (3.8), it is easy to obtain the estimate

$$I_2 \sim 2\int_{\xi}^{\infty} I_1(\xi_1)\,d\xi_1,$$

leading to the conclusion that I_2 decays exponentially when $\xi \to \infty$. The issue of the convergence of J_3 is somewhat more complicated. Actually, the application of Eq. (3.13) to Eq. (3.12) yields

$$J_1 \sim J_{10} + 4\ln(\xi_2 - b) + \ldots$$

with a constant J_{10}. The observed divergence of J_1 at $\xi_2 \to +\infty$ is weak, so the integral J_2, determined through J_1 by means of (3.11), not only converges, but exponentially approaches zero at $\xi_1 \to \infty$. Now from (3.10) we obtain the following estimate for J_3

$$J_3 \sim 2\int_{\xi}^{\infty} J_2(\xi_1)\,d\xi_1 + \ldots,$$

guaranteeing exponential decay of the quantity when $\xi \to \infty$.

Since the relation between the variables ξ and η in (3.5) is linear, boundary condition (3.3) can be satisfied only if we set the constant $A_2 = 0$ in solution (3.7). As regards the constants A_1 and A_3, they remain arbitrary for now.

Having inserted the asymptotic representation (3.14) of the Champion function into Eq. (3.9), when $\xi_1 \to -\infty$ we find

$$I_1 \sim I_{10} - c^{-3}\xi_1 + \ldots,$$

where I_{10} is a constant. Then from (3.8) we have

$$I_2 \sim \frac{1}{cC}\left(\frac{c^4 I_{10} - 1}{c} - \xi\right) e^{-c\xi} + \ldots, \tag{3.15}$$

if we limit ourselves to the terms which increase exponentially when $\xi \to -\infty$. Being used to calculate integral (3.12), asymptote (3.14) gives

$$J_1 \sim J_{10} - c\xi_2 + \ldots,$$

when $\xi_2 \to -\infty$, where J_{10} is a constant. In accordance with (3.11), the increase in J_2 when $\xi_1 \to -\infty$ also occurs algebraically

$$J_2 \sim J_{20} - \frac{1}{c^3}\left(J_{10}\xi - \frac{c}{2}\xi^2\right) + \ldots$$

with yet another constant J_{20}. Retaining in the right side of (3.10) only the terms which increase exponentially when $\xi \to -\infty$, we write

$$J_3 \sim \frac{1}{c}\left(c^2 J_{20} + \frac{1 - J_{10}}{c^2} + \frac{1 - J_{10}}{c}\xi + \frac{1}{2}\xi^2\right) e^{-c\xi} + \ldots \tag{3.16}$$

Equations (3.15) and (3.16) make it possible to obtain an asymptotic expansion of solution (3.7) in the form

$$f \sim -c + \frac{C}{c^2} e^{c\xi} + \frac{\varepsilon A_3}{c^2}\left(\frac{c^4 I_{10} - 1}{c} - \xi\right) + \\ + \frac{\beta_0}{c}\left(c^2 J_{20} + \frac{1 - J_{10}}{c^2} + \frac{1 - J_{10}}{c}\xi + \frac{1}{2}\xi^2\right) + \ldots \tag{3.17}$$

4. The above-constructed linear approximation is not uniformly accurate when $\xi \to -\infty$. It can be modified in this region in the following manner. We introduce the small parameter $\alpha = \varepsilon A_3/c^2$ and assume that the ratio $|\beta_0|/\alpha \ll 1$. It is understood that this assumption requires validation during the course of the subsequent analysis. Now we leave the following dominant terms in asymptotic expansion (3.17)

$$f \sim -c - \alpha(d + \xi) + \ldots, \quad d = \frac{1 - c^4 I_{10}}{c} \tag{4.1}$$

and, recalling Eq. (3.6), we simplify Eq. (2.1). As a result

$$\frac{d^3 f}{d\xi^3} - [c + \alpha(d + \xi)]\frac{d^2 f}{d\xi^2} + \beta_0 = 0.$$

From here we obtain an expression for the second derivative [6]

$$\frac{d^2 f}{d\xi^2} = e^{\frac{1}{2\alpha}[c + \alpha(d + \xi)]^2}\left\{D - \beta_0 \int_0^\xi e^{-\frac{1}{2\alpha}[c + \alpha(d + \xi_1)]^2} d\xi_1\right\}. \tag{4.2}$$

The arbitrary constant D in this expression is determined by joining (4.2) with the asymptote

$$\frac{d^2 f}{d\xi^2} \sim C e^{c\xi} + \frac{\beta_0}{c} + \ldots,$$

following from (3.17). This requirement is satisfied if

$$D = C e^{-c^2/2\alpha - cd}. \tag{4.3}$$

We will satisfy the first of conditions (1.5) or (1.6), applying both to flow in a wake and for the boundary layer on the plate. In accordance with (3.5), the variable $\xi = -a$ at $\eta = 0$. Having used approximate representation (4.1), we find

$$a = c/\alpha + d, \tag{4.4}$$

from which we immediately conclude that

$$\frac{d^2 f(0)}{d\eta^2} = D + \beta_0 \sqrt{\frac{\pi}{2\alpha}}. \tag{4.5}$$

Integrating of Eq. (4.2) yields

$$\frac{df}{d\xi} = \int_0^\xi e^{\frac{1}{2\alpha}[c + \alpha(d + \xi_1)]^2}\left\{D - \beta_0 \int_0^{\xi_1} e^{-\frac{1}{2\alpha}[c + \alpha(d + \xi_2)]^2} d\xi_2\right\} d\xi_1 + E, \tag{4.6}$$

while the arbitrary constant E should be chosen by combination with the asymptotic expansion

$$\frac{df}{d\xi}=\frac{C}{c}e^{c\xi}-\alpha+\frac{\beta_0}{c}\left(\frac{1-J_{10}}{c}+\xi\right)+\ldots,$$

which follows from (3.17). The union is achieved when

$$E=\frac{C}{c+\alpha d}-\alpha-\frac{\beta_0 J_{10}}{c^2}. \tag{4.7}$$

With allowance for Eq. (4.4), from here we have

$$\frac{df(0)}{d\eta}=-\alpha-\beta_0\left\{\frac{J_{10}}{c}-\int\limits_{-\left(\frac{c}{\alpha}+d\right)}^{0}e^{\frac{1}{2\alpha}[c+\alpha(d+\xi_1)]^2}\int\limits_0^{\xi_1}e^{-\frac{1}{2\alpha}[c+\alpha(d+\xi_2)]^2}d\xi_2 d\xi_1\right\}.$$

Definitions (4.3) and (4.7) of constants D and E are equally valid for both classes of flows examined here.

5. To construct the velocity field in the symmetrical wake, it is necessary to satisfy the second condition of (1.5). From (4.5) we find the equality

$$\beta_0=-D\sqrt{\frac{2\alpha}{\pi}}=-C\sqrt{\frac{2\alpha}{\pi}}\,e^{-c^2/2\alpha-cd}, \tag{5.1}$$

first presented in [6].

We make an estimate of the double integral from the right side of Eq. (4.8). For convenience, we represent it as $-I$, since $I < 0$. The folowing inequalities exist

$$\frac{1}{\xi+\sqrt{\xi^2+2}}<e^{\xi^2}\int\limits_{\xi}^{\infty}e^{-\xi_1^2}d\xi_1\leqslant\frac{1}{\xi+\sqrt{\xi^2+4/\pi}},$$

the use of which makes it possible to confirm that

$$\frac{1}{\alpha}\left(\ln\frac{c}{\sqrt{2\alpha}}+\frac{1}{2}+\frac{1}{2}\ln 2\right)<-I\leqslant\frac{1}{\alpha}\left(\ln\frac{c}{\sqrt{2\alpha}}+\frac{1}{2}+\frac{1}{2}\ln\pi\right). \tag{5.2}$$

Despite the fact that this integral increases without limit when $\alpha\to 0$, its contribution to Eq. (4.8) is negligible if we are studying the velocity field in the wake. This is because the coefficient β_0 exponentially approaches zero in this case as a result of (5.1). Thus, the small parameter [6] $\alpha = -df(0)/d\eta$.

The circumstances are different in the case of flow with a boundary layer on a plate, when Eq. (4.8) itself must vanish. Instead of Eq. (5.1) we have the inequalities

$$-\frac{\alpha^2}{\ln\frac{c}{\sqrt{2\alpha}}+\frac{1}{2}+\frac{1}{2}\ln 2}<\beta_0\leqslant-\frac{\alpha^2}{\ln\frac{c}{\sqrt{2\alpha}}+\frac{1}{2}+\frac{1}{2}\ln\pi} \tag{5.3}$$

based on (5.2), where

$$\frac{\beta_0}{\sqrt{\alpha}}=\sqrt{\frac{2}{\pi}}\frac{d^2f(0)}{d\eta^2}.$$

As relations (5.1) and (5.3) show, $\beta_0 < 0$, while $|\beta_0|/\alpha \ll 1$ in accordance with the assumption made above. Thus, even very small changes in pressure on the outside edge of the boundary layer can have a significant effect on the motion of the fluid in the recirculation region. This fact explains the results from [5] discussed in Part 2.

Let us look at the main properties of the velocity fields constructed here. First of all, we find from Eqs. (3.5) and (4.4) that

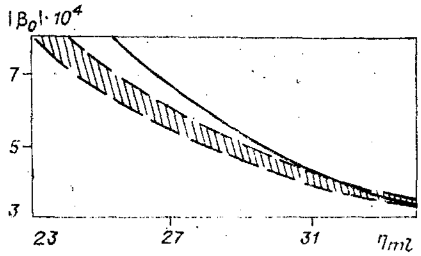

Fig. 2

$$\eta = \eta_{ml} = c/\alpha + d \tag{5.4}$$

when $\xi = 0$. It follows from this that the mixing layer moves away from the symmetry axis of the wake or plate with a decrease in the pressure gradient, characterized by the parameter β_0. This conclusion from the asymptotic analysis confirms the results of the calculations summed up in Fig. 1.

However, there is a substantial difference between the two classes of flows investigated here. It is easily seen that given the same α, the value of $|\beta_0|$ obtained in accordance with (5.1) is an order of magnitude less than the value obtained from inequalities (5.3). Conversely, with a prescribed value of β_0, the value of the parameter α calculated from (5.1) is much greater than that lying within the range of (5.3). In sum, given the same gradient of external pressure, the mixing layer bounding the region of recirculatory motion in the wake is located considerably below the analogous layer formed in flow about the plate. This conclusion – which is also consistent with the calculated data in Fig. 1 – contradicts inductive representations. According to the latter, the presence of a thin body in a flow significantly changes the velocity field only near its surface. It is clear from this that the introduction, by the experimenters in [8, 9], of a so-called separating plate in the wake of a bluff body in order to damp axisymmetric oscillations may be accompanied by radical restructuring of the entire flow. Correct interpretation of the results of such tests demands that the separating plate be regarded as an integral part of the obstacle in the flow.

Viscous shear stresses determine the structure of the overall velocity field. In fact, the mixing layer and its neighborhood – embracing the outside edge of the recirculation region – are described by variation (3.1) of the Champion function. Equations (4.2) and (4.6) are valid for the internal part of the region of reverse flow. The third derivative $d^3f/d\eta^3$ in the initial Falkner–Skan equation also played an important role in the derivation of Eqs. (4.2) and (4.6). Moreover, without resorting to these formulas, it would be impossible to satisfy the boundary conditions either on the surface of the body or on the symmetry axis of the wake without losing the connection between the pressure gradient and the position of the mixing layer.

In conclusion, let us compare the thickness of the reverse stream obtained from the asymptotic analysis with the thickness with the tickness found by direct numerical integration of the Falkner–Skan equation. Having subjected the Champion function to the additional condition [6] that the constant $C = c^3$ in expansion (3.14) when $\xi \to -\infty$, for the sake of convenience we drop the condition $df_0/d\xi = 0$, with $\xi = 0$, when determining the function. Then $c = 0.876$ and $d = 1.765/c = 2.014$. These values fix the values of β_0 and $\eta_{m\ell}$ in Eqs. (5.1), (5.3), and (5.4). The results of the calculations are shown in Fig. 2, where the region between the dashed lines was constructed from (5.3) and the solid curve represents data from numerical integration of Eq. (2.1) with different β_0 for the boundary layer on the plate.

LITERATURE CITED

1. G. Shchlichting, Boundary Layer Theory [in Russian], Nauka, Moscow (1974).
2. D. R. Chapman, Laminar Mixing of a Compressible Fluid. Rept/NACA (1950), No. 958.
3. K. Stewartson, "Further solutions of the Falkner–Skan equation," Proc. Cambridge Philos. Soc., 50, Pt. 3 (1954).
4. E. D. Kennedy, "Wave-like solutions fo the laminar boundary layer equations," AIAA J., 2, No. 2 (1964).
5. F. T. Smith, "Non-uniqueness in wakes and boundary layers," Proc. R. Soc. A, 391, No. 1800 (1984).

6. K. Stewartson, "Falkner—Skan equation for wakes," AIAA J., 2, No. 7 (1964).
7. M. Abramovits and I. Stegun (eds.), Handbook of Special Functions [Russian translation], Nauka, Moscow (1979).
8. A. Acrivos, D. D. Snowden, et al., "The steady separated flow past a circular cylinder at large Reynolds numbers," J. Fluid Mech., 21, Pt. 4 (1965).
9. A. Acrivos, L. G. Leal, et al., "Further experiments on steady separated flows past bluff objects," J. Fluid Mech., 34, Pt. 1 (1968).

HEAT TRANSFER IN A FILM FLOWING OVER THE SURFACE OF A CONVERGENT DUCT

A. M. Lapin, L. I. Sen', and A. M. Te

UDC 621.181.61:629.12

Efficient organization of the process of thermal softening of highly-mineralized natural liquids such as sea water requires their heating to temperatures above 200°C with a nonboiling regime of operation of the heat-transfer unit. One method of realizing such heating is the use of film-type units, in which heat is supplied to a laminar film of liquid from the free phase boundary [1]. In contrast to recuperative heat exchange, in this case the minearalized liquid can be heated to a high temperature while the temperature of the boundary layer of the film is relatively low. It is this circumstance that permits nonboiling operation of the water heater.

It has been established experimentally [2] that the flow of fluid in a convergent duct with a total convergence angle of more than 90° (in contrast to flow over a vertical surface) results in a two-dimensional laminar nonwavy regime of film flow with a broad range of flow rates. This hydrodynamic feature makes it possible to more fully utilize the advantages of the given method of heating and accounts for the preference of using convergent-duct-film units [3, 4] to heat scale-forming solutions.

Here we study the process of contact heat exchange in the condensation of pure vapor on a film of liquid flowing over the surface of a convergent duct.

Formulation of the Problem. Assuming the problem to be steady and axisymmetric, we write the following equations of conservation of momentum, continuity, and energy in a boundary-layer approximation for a thin liquid film:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)+g\sin\alpha; \tag{1}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y}+g\cos\alpha=0; \tag{2}$$

$$\frac{\partial (ru)}{\partial x}+\frac{\partial (rv)}{\partial y}=0; \tag{3}$$

$$\rho c\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right). \tag{4}$$

Here, x is the longitudinal coordinate, directed downflow along the generatrix of the convergent duct; y is the transverse coordinate, directed perpendicular to the generatrix of the duct; the origin of the coordinates is on the inlet edge of the duct; u and v are respectively the x and y components of velocity; g is acceleration due to gravity; p is pressure; α is the angle of inclination of the duct generatrix to the horizontal; r(x) is the running radius of the duct; ν, ρ, c, λ, and T are the kinematic viscosity, density, specific heat, thermal conductivity, and temperature of the liquid.

The problem is solved with the following assumptions: the subjacent surface of the duct is thermally insulated; there is no shear stress on the liquid-vapor boundary; the

Vladivostok. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 54-58, May-June, 1986. Original article submitted March 25, 1985.

 0021-8944/86/2703-0366$12.50